XXXVI. *On the Preceſſion of the Equinoxes.* By the Rev. Samuel Vince, *M. A. F. R. S.*

Read June 21, 1787.

1. THE true cauſe of the preceſſion of the equinoctial points was firſt aſſigned by Sir ISAAC NEWTON; but it is confeſſed, that he has fallen into an error in his inveſtigation of the effect. Without, however, entering into any enquiry relative to the circumſtances in which he has erred, I propoſe to ſhew how we may obtain a true ſolution from his own principles, by means of which alone the whole calculation may be rendered extremely ſimple and evident: and although very ſatisfactory ſolutions have been already given, yet the importance of the problem will ſufficiently apologiſe for offering any thing further upon the ſubject that may at all tend to elucidate it.

2. Let S (Tab. XV. fig. 1.) be the ſun, ABDC the earth, T its center, EQ the equator, P, *p*, the poles; draw CTB perpendicular to SAD, and join SE, which produce to meet CB in K. Call the radius ET unity, and let the force of the ſun on a particle at T be $\frac{1}{ST^2}$, then the force on a particle at E = $\frac{1}{SE^2}$; hence, if we reſolve this latter force into two others, one in the direction ET, and the other in the direction parallel to TS, we have SE : ST :: $\frac{1}{SE^2}$: the force in the direction parallel to

to $TS = \frac{ST}{SE^3} = \frac{ST}{\overline{ST - EK}^3} = \frac{1}{ST^2} + \frac{3EK}{ST^3}$, omitting the other terms of the ſeries on account of their ſmallneſs. Hence the force with which a particle at E is drawn *from* CB is equal to $\frac{3EK}{ST^3}$; conſequently the effect of this force in a direction perpendicular to ET will be $\frac{3EK \times KT}{ST^3}$; hence this force : the force of the ſun on a particle at T :: $\frac{3EK \times KT}{ST^3} : \frac{1}{ST^2} :: 3EK \times KT : ST$. Now if $P =$ the periodic time of the earth, $p =$ the periodic time of a body revolving at the earth's ſurface; then the force of the earth to the ſun : the force of the body to the earth, or the force of gravity, :: $\frac{ST}{P^2} : \frac{1}{p^2}$; and hence the force of the ſun on a particle at E perpendicular to ET : the force of gravity :: $\frac{3EK \times KT \times p^2}{P^2} : 1$.

3. Let v be the center of gyration, and put $M =$ the quantity of matter in the earth: then the effect of the inertia of M placed at v, to oppoſe the communication of motion, is the ſame as the effect of the inertia of the earth; and hence, by the property of that center, $ET^2 : Tv^2 \left(= \frac{2}{5} ET^2\right) :: M : \frac{2}{5} M$, which is the quantity of matter to be placed at E to have the ſame effect.

4. Put $m =$ the exceſs of the quantity of matter in the earth above that of its inſcribed ſphere. Now by Sir ISAAC NEWTON's two firſt lemmas, it appears, that the action of the ſun upon the ſhell m of matter, to generate an angular velocity about an axis perpendicular to CABD, is juſt the ſame as it would be to generate an angular velocity in a quantity of matter equal to $\frac{1}{5} m$ placed at E. Let us therefore ſuppoſe the ſun's attraction, perpendicular to ET, to be exerted upon a

quantity of matter at E equal to $\frac{1}{5}m$, and at the ſame time to have a quantity of matter to move equal to $\frac{2}{5}$M, and then from this and art. 3. it appears, that the effect will be the ſame as the accelerative force of the ſun to turn about the earth. Hence that accelerative force is, from art. 2. equal to $\frac{3\mathrm{EK}\times\mathrm{KT}\times p^2\times\frac{1}{5}m}{\frac{2}{5}\mathrm{M}\times\mathrm{P}^2}=\frac{3\mathrm{EK}\times\mathrm{KT}\times p^2\times m}{2\mathrm{M}\times\mathrm{P}^2}$, gravity being unity. Now, if TE : TP :: $1 : 1-r$, then $\mathrm{M} : \mathrm{M}-m :: 1 : 1-2r$, therefore, $\mathrm{M} : m :: 1 : 2r$, hence $\frac{m}{2\mathrm{M}}=r$, conſequently the accelerative force $=\frac{3\mathrm{EK}\times\mathrm{KT}\times p^2\times r}{\mathrm{P}^2}$.

5. Let $\dot{z}=$ the arc deſcribed by a point of the equator about its axis in an indefinitely ſmall given time, which may therefore repreſent its velocity; and let $a\dot{z}$ repreſent the arc deſcribed in the ſame time by a body revolving about the earth at its ſurface; then $\frac{a^2\dot{z}^2}{2}=$ the ſagitta of the arc deſcribed by the body in the ſame time, and conſequently $a^2\dot{z}^2=$ the velocity generated by gravity whilſt a point of the equator deſcribes $\dot{z}$. Hence, by art. 4. we have $1 : \frac{3\mathrm{EK}\times\mathrm{KT}\times p^2\times r}{\mathrm{P}^2} :: a^2\dot{z}^2 : \frac{3\mathrm{EK}\times\mathrm{KT}\times p^2\times r\times a^2\dot{z}^2}{\mathrm{P}^2}$ the velocity of the point E of the equator generated by the action of the ſun, whilſt the equator deſcribes $\dot{z}$ about its axis; conſequently the ratio of theſe velocities is as $\frac{3\mathrm{EK}\times\mathrm{KT}\times p^2\times a^2\dot{z}}{\mathrm{P}^2} : 1$.

6. Let $\dot{y}$ be an arc deſcribed by the ſun in the ecliptic to a radius equal to unity, whilſt a point of the equator deſcribes $\dot{z}$ about its axis; then (as $ap=$ the time of the earth's rotation, and the arcs deſcribed in equal times to equal radii are inverſely as the periodic times) $\frac{1}{\mathrm{P}} : \frac{1}{ap} :: \dot{y} : \dot{z}=\frac{\mathrm{P}\dot{y}}{ap}$; hence, if v and w be put

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for the ſine and coſine of the ſun's declination, the ratio of the velocities in the laſt article becomes $\frac{3aprvw\dot{y}}{\text{P}}$: 1.

7. Hence if TSL (fig. 2.) be the ecliptic to the radius unity, P the plane of the ſun, SER the equator, PE the ſun's declination, and we take E*c* : *cd* (*cd* being perpendicular to E*c*) :: 1 : $\frac{3aprvw\dot{y}}{\text{P}}$, and through *d*, E, deſcribe the great circle TEM, then will ST be the preceſſion of the equinox during the time the ſun deſcribes $\dot{y}$ in the ecliptic. Now E*d* (or E*c*, as the angle at E is indefinitely ſmall) : *dc* :: rad. $=1$: ſine angle $\text{E}=\frac{3aprvw\dot{y}}{\text{P}}$; hence (if SV be drawn perpendicular to TE) 1 : ſine SE :: $\frac{3aprvw\dot{y}}{\text{P}}$: $\text{SV}=\frac{3aprvw\times\text{ſin. SE}\times\dot{y}}{\text{P}}$; therefore, ſin. STV or ESP : 1 :: SV : $\text{ST}=\frac{3aprvw\times\text{ſin. SE}\times\dot{y}}{\text{P}\times\text{ſin. ESP}}$.

8. Now $\frac{v}{\text{ſin. ESP}}=\text{ſin. SP}$, and $w=\frac{\text{coſ. SP}}{\text{coſ. ES}}$, hence $\frac{vw}{\text{ſin. ESP}}=\frac{\text{ſin. SP}\times\text{coſ. SP}}{\text{coſ. ES}}$; but $\frac{\text{coſ. ESP}}{\text{tan. ES}\times\text{cot. SP}}=1$, hence $\frac{vw}{\text{ſin. ESP}}=\frac{\text{ſin. SP}\times\text{coſ. SP}\times\text{coſ. ESP}}{\text{coſ. ES}\times\text{tan. ES}\times\text{cot. SP}}=\frac{\overline{\text{ſin. SP}}^2\times\text{coſ. ESP}}{\text{ſin. ES}}$; conſequently,

$$\text{ST}=\frac{3apr\times\overline{\text{ſin. SP}}^2\times\text{coſ. ESP}\times\dot{y}}{\text{P}}=(\text{if } x=\text{ſin. SP})$$

$\frac{3apr\times\text{coſ. ESP}\times x^2\dot{x}}{\text{P}\times\sqrt{1-x^2}}$, whoſe fluent, when $x=1$, is $\frac{3apr\times\text{coſ. ESP}\times y}{2\text{P}}$ (y being now = to a quadrant) the arc of preceſſion whilſt the ſun deſcribes 90° of the ecliptic; and to find the degrees ſay, $4y : 360^\circ :: \frac{3apr\times\text{coſ. ESP}\times y}{2\text{P}} : 360^\circ\times\frac{3apr\times\text{coſ. ESP}}{8\text{P}}$, conſequently the preceſſion in a year $=360^\circ\times\frac{3apr\times\text{coſ. ESP}}{2\text{P}}=21''\ 6'''$. This would be the preceſſion of the equinox ariſing from the attraction of the ſun, if the earth were of an uniform denſity, and the ratio of

Fig. 1.

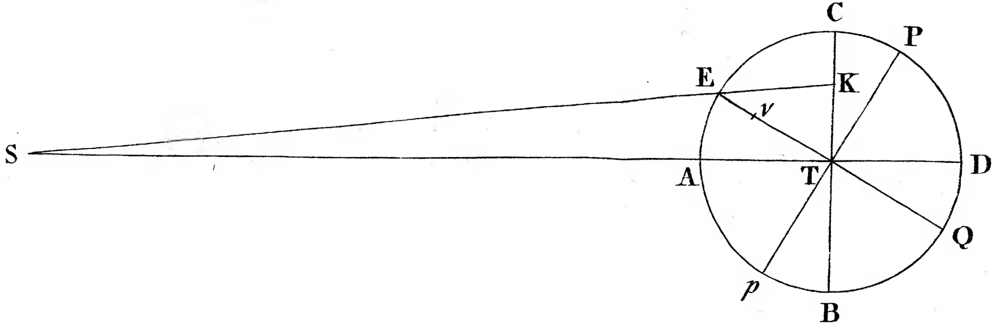
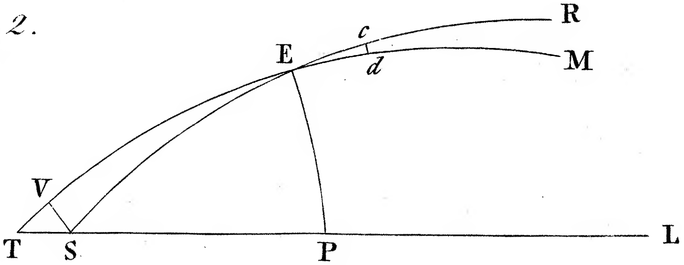


Fig. 2.



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of the diameters as 229 : 230; but if the greateſt nutation of the earth's axis be rightly aſcertained, the preceſſion is only about $14''\frac{2}{3}$; which difference between the theory and what is deduced from obſervation muſt ariſe either from the fluidity of the earth's ſurface, an increaſe of denſity towards the center, or the ratio of the diameters being different from what is here aſſumed, or probably from all the cauſes conjointly. But as the beſt obſervations muſt be liable to ſome ſmall degree of inaccuracy, and an error of one or two ſeconds in the nutation will, in this caſe, make a very conſiderable alteration in the concluſion, the eſtimation of the preceſſion ariſing from the action of the ſun ſeems to be ſubject to a very conſiderable degree of uncertainty.